B.SC. THIRD SEMESTER (HONS.) EXAMINATION 2021

Subject: Mathematics **Course ID: 32114**

Course Title: Algebra **Course Code: SH/MTH/304/GE-3**

Full Marks: 40 **Time: 2 hour**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning.

1. **Answer *any five* of the following questions:** **(2x5=10)**

a) Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots+\frac{1}{n.(n+1)}=\frac{n}{n+1}$.

b) Prove that $\frac{(n+1)^n}{2^n}>n!$.

c) Show that the product of all values of $(\sqrt{3}+i)^{\frac{3}{5}}$ is $8i$.

d) Apply Descartes' rule of sign to examine the nature of roots of the equation

$$x^4-4x^3+2x^2+4x+1=0.$$

e) What is the smallest equivalence relation on the set $A=\{a,b,c\}$.

f) Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=|x|+x$ is neither injective nor surjective.

g) Find the dimension of the subspace W of$\mathbb{R}^3$ defined by
$W=\{(x,y,z)\in\mathbb{R}^3: 2x+y-z=0\}$.

h) Use Cayley-Hamilton theorem to compute A^{2020}, where $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$.

2. **Answer *any four* of the following questions:** **(5x4=20)**

a) If $(1+x)^n=a_0+a_1x+a_2x^2+\cdots$, prove that
$a_0+a_4+a_8+\cdots=2^{n-2}+2^{\frac{n-2}{2}}cos\frac{n\pi}{4}$.

b) If one root of the equation $x^3+ax+b=0$ is twice the difference of the other two, then prove that one root is $\frac{13b}{3a}$.

c) (i) Show that $1!.3!.5!\ldots..(2n-1)!>(n!)^n$.

(ii) Prove that the number of primes is infinite. $3+2$

d) Determine the conditions of a and b for which the system of equations has (i) only one solution, (ii) no solution, (iii) many solutions:

$$x+2y+z=1$$
$$2x+y+3z=b$$
$$x+ay+3z=1+b.$$

e) A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$T(x, y, z) = (x + y + z, 2x + y + 2z, x + 2y + z), (x, y, z) \in \mathbb{R}^3$.

Show that T is a linear mapping. Find $Ker\ T$ and the dimension of $Ker\ T$.

f) Find the real eigen values (if any) and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}.$$

3. Answer *any one* of the following questions: (1x10=10)

a) (i) Let z be a variable complex number such that the amplitude of $\frac{Z-i}{Z+1}$ is $\frac{\pi}{4}$. Show that the point z lies on a circle in the complex plane.

(ii) Prove that the product of any m consecutive integers is divisible by m.

(iii) Show that the mapping $f: S \to \mathbb{R}$ defined by $f(x) = \frac{x}{1-|x|}$, where $S = (-1, 1)$, is bijective. 4+3+3

b) (i) Prove that the eigen values of a real symmetric matrix are all real.

(ii) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis vectors $(0,1,1), (1,0,1), (1,1,0)$ of $\mathbb{R}^3$ to the vectors $(2,1,1), (1,2,1), (1,1,2)$ of $\mathbb{R}^3$ respectively. Also find $Ker\ T$ and $Im\ T$. 5+5

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